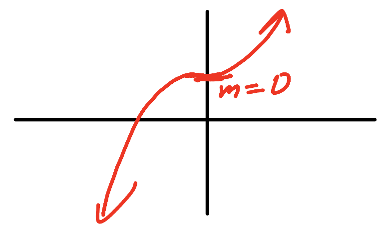
4. Each of the following statements is not always true. Explain/Show why each statement could be false. Tip: One contradiction is enough.

a) If $f'(5) = 0$, then there is a maximum or minimum at $x = 5$.



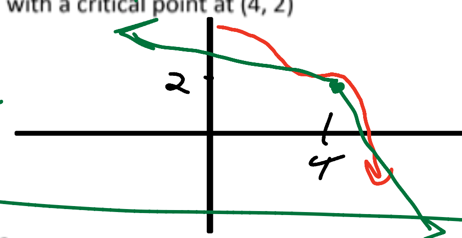
b) If $x = 2$ is a critical point, then $f'(2) = 0$.

c) An extrema occurs at every critical point

d) If m is a local minimum and M is a local maximum of a continuous function, then $m < M$.

$F'(4)=0$ or $\emptyset$

5. (Multiple Choice) If f is a continuous, decreasing function on [0, 10] with a critical point at (4, 2) which of the following statements must be false?



A) $f(10)$ is an absolute minimum of f on [0, 10]. True

B) $f(4)$ is neither a relative maximum nor a relative minimum True

C) $f'(4)$ does not exist False $F'(4)=0$ maybe

D) $f'(4) = 0$ False $F'(4)=\emptyset$

E) $f'(4) < 0$ → False!!!

6. Find the equation of the tangent line to the graph of $y^3 + (xy + 2)^2 = 0$ at the point (3, −1)

$$3y^2\frac{dy}{dx} + 2(xy+2)^1\left(1\cdot y + x\cdot\frac{dy}{dx} + 0\right) = 0$$

$$3(-1)^2\frac{dy}{dx} + 2(3\cdot -1+2)^1\left(-1 + 3\cdot\frac{dy}{dx}\right) = 0$$

$$3\frac{dy}{dx} + -2\left(-1 + 3\frac{dy}{dx}\right) = 0$$

$$3\frac{dy}{dx} + 2 - 6\frac{dy}{dx} = 0$$

$$2 - 3\frac{dy}{dx} = 0$$

$$\frac{2}{3} = \frac{dy}{dx}$$

$m = \frac{2}{3}$ $(+3, -1)$

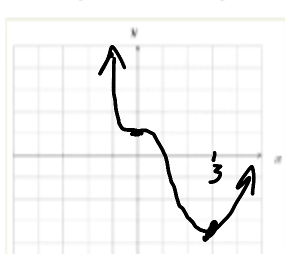
$$y - (-1) = \frac{2}{3}(x-3)$$

$$y + 1 = \frac{2}{3}x - 2$$

$-1 \qquad -1$

$$\boxed{y = \frac{2}{3}x - 3}$$

3. Sketch a function with critical number (no extrema) at $x = 0$ and absolute minimum at $x = 3$.



d) $y = -x^3 + 2x^2 + 1$ over $[-1,1]$

$\frac{dy}{dx} = -3x^2 + 4x = x(-3x+4) = 0$ or $\emptyset$

$x=0$ or $-3x+4=0$

$-3x = -4$

$x = \frac{-4}{-3} = 1\frac{1}{3}$ ← Not on Interval

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b) $f(x) = \sin x - \cos^2 x$ with $0 < x < 2\pi$.

$F'(x) = \cos x - 2(-\sin x)(\cos x)$

$F'(x) = \cos x + 2\sin x \cos x = 0$ or $\emptyset$

$F'(x) = \cos x(1 + 2\sin x)$

$\cos x = 0$ or $1 + 2\sin x = 0$

$\cos x = 0$ or $\sin x = -\frac{1}{2}$

$x = \frac{\pi}{2}, \frac{3\pi}{2}$

$\frac{7\pi}{6}, \frac{11\pi}{6}$

$y = \cos^2 x \Rightarrow y = u^2$

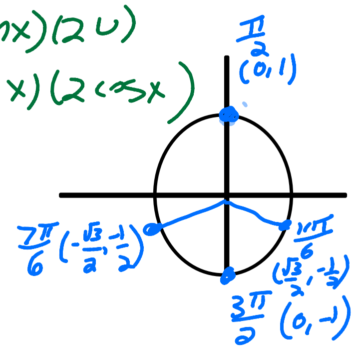
$u = \cos x$

$\frac{dy}{du} = 2u$

$\frac{du}{dx} = -\sin x$

$(-\sin x)(2u)$

$(-\sin x)(2\cos x)$



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c) $y = \sqrt[3]{x}$ over $[-8,8]$

$y = x^{\frac{1}{3}}$

$\frac{dy}{dx} = \frac{1}{3}x^{\frac{1}{3}-1=\frac{-2}{3}} = \frac{1}{3}x^{-\frac{2}{3}} = \frac{1}{3\sqrt[3]{x^2}} = \emptyset$

$3\sqrt[3]{x^2} = 0$

$x = 0$

Rolles

Example 1: Find the two x-intercepts of $f(x) = x^2 - 3x + 2$ and show that $f'(x) = 0$ at some point between the two x-intercepts.

$F(x) = x^2 - 3x + 2 = (x-2)(x-1) = 0$

$x = 1$ and $x = 2$

$F(1) = 0 = 1 - 3 + 2$

$F(2) = 0 = 4 - 6 + 2$

$[1, 2]$

$1 < \frac{3}{2} < 2$

$F'(x) = 2x - 3$

$0 = 2x - 3$

$+3 \quad +3$

$3 = 2x$

$\frac{3}{2} = x$

works

Example 2: Let $f(x) = \cos 2x$. Find all values of c in the interval $[-\pi, \pi]$ such that $f'(c) = 0$.

$F(\pi) = \cos 2\pi = 1$

$F(-\pi) = \cos -2\pi = 1$

$F(\pi) = F(-\pi)$

$x = \frac{\pi}{2}, -\frac{\pi}{2}, 0$

$F'(x) = (-\sin 2x)2$

$F'(x) = -2\sin 2x$

$\frac{\cancel{-2}\sin 2x}{\cancel{-2}} = \frac{0}{-2}$

$\sin 2x = 0$

$2\sin x \cos x = 0$

$\cos x = 0$

$\frac{\pi}{2}, \frac{3\pi}{2}$

$-\frac{\pi}{2}, -\frac{3\pi}{2}$

$\sin x = 0$

$0, \pi, 2\pi, -\pi$

Student Example: Let $f(x) = x^4 - 2x^2$. Find all values of c in the interval [-2, 2] such that $f'(c) = 0$.

$F(-2) = (-2)^4 - 2(-2)^2 = 16 - 8 = 8$

$F(2) = 2^4 - 2(2)^2 = 16 - 8 = 8$

$F(-2) = F(2)$

$F'(x) = 4x^3 - 4x = 4x(x^2 - 1) = 4x(x-1)(x+1) = 0$

$4x = 0$ or $x - 1 = 0$ or $x + 1 = 0$

$x = 0$ or $x = 1$ or $x = -1$

Example 2.1:

$f(x) = x^2 + 2$ on [-2,2]

$F'(x) = 2x$

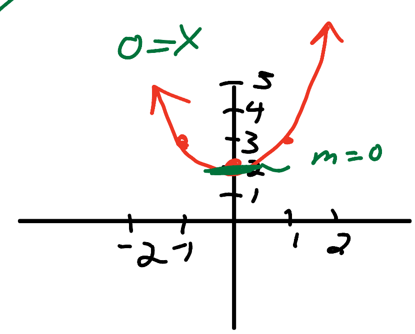
$0 = 2x$

$0 = x$

$F(-2) = (-2)^2 + 2 = 4 + 2 = 6$

$F(2) = (2)^2 + 2 = 4 + 2 = 6$

$$\frac{F(-2) - F(2)}{-2 - 2} = \frac{6-6}{-4} = \frac{0}{-4} = 0$$



Example 3: Given $f(x) = 5 - \frac{4}{x}$, find all values of c in the closed interval [1, 4] such that $f'(c) = \frac{f(4)-f(1)}{4-1}$. $= \frac{4-1}{3} = \frac{3}{3} = 1$

$F(1) = 5 - \frac{4}{1} = 5 - 4 = 1$

$F(x) = 5 - 4x^{-1}$

$F(1) = 1$

$F'(x) = 0 - 4(-1)x^{-1-1} = 4x^{-2} = \frac{4}{x^2}$

$F(4) = 5 - \frac{4}{4} = 5 - 1 = 4$

$F(4) = 4$

$1 = \frac{4}{x^2} \Rightarrow x^2 = 4$

$x = \pm 2$

$(1,1)\ (4,4)$

interval $[1,4]$

$x = 2$

$\frac{-3}{-3} = \frac{1-4}{1-4} = \frac{4-1}{4-1} = \frac{3}{3} = \underline{1}$

Example 4: Verify that the function $f(t) = t^3 - 3t + 5$, for $-1 \le t \le 1$ satisfies the conditions for the Mean Value Theorem. Find the number(s) c.

$F(-1) = (-1)^3 - 3(-1) + 5 = 7 + 3 + 5 = 7$ $(-1, 7)$

$F(1) = 1^3 - 3(1) + 5 = 1 - 3 + 5 = 3$ $(1, 3)$

$F'(T) = 3T^2 - 3$

$\frac{F(-1) - F(1)}{-1 - 1} = \frac{7-3}{-2} = \frac{4}{-2} = -2$

$F'(T) = -2$

$-2 = 3T^2 \cancel{-3}$

$+3 \qquad \cancel{+3}$

$1 = 3T^2$

$\pm\frac{\sqrt{3}}{3} = \pm\frac{1}{\sqrt{3}} = T$

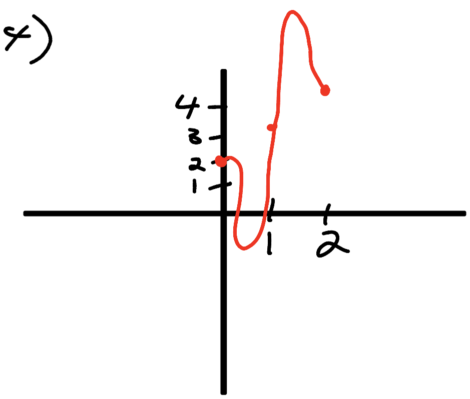
5 miles in 4 min $= \frac{1}{15}$ hour

$Dis = R \cdot T$

$15 \cdot 5 = R \cdot \frac{1}{\cancel{15}} \text{hour} \cdot \cancel{15}$

75 mph = Rate

(0,2)(1,3)(2,4)



If $h(x) = f(x)g(x)$ and $h'(x) = f'(x)g(x)$ for all real numbers, then $g(x) =$

$h'(x) = F'(x) \cdot g(x) + \underbrace{F(x) \cdot g'(x)}_{\text{missing}}$

$g'(x) = 0$